

DCS/CSCI 2350: Social & Economic Networks

Matching

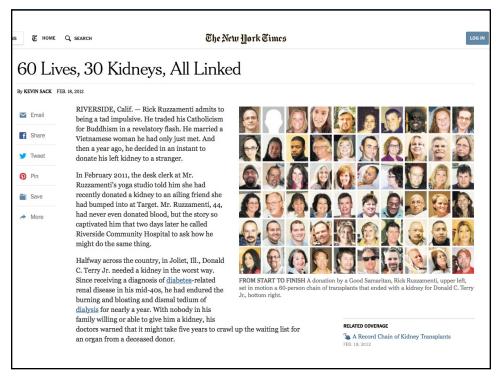
Mohammad T. Irfan

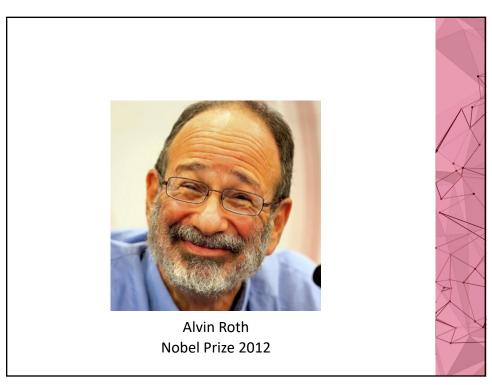
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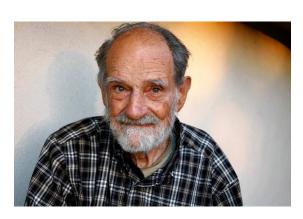


Two-sided matching

Handout for stable marriage (on Canvas)







Lloyd Shapley Nobel Prize 2012

Stable matching

Given n men and n women, where each man ranks all women and each woman ranks all men, find a <u>stable</u> <u>matching</u>.

<u>Stable matching:</u> There exists no unmatched pair (X, Y) that prefer each other over their matched partners.

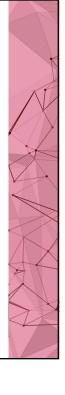
• Such (X, Y) pair if exists is called a "blocking pair"



Perfect matching

Each person is matched to another

Necessary condition: # men = # women



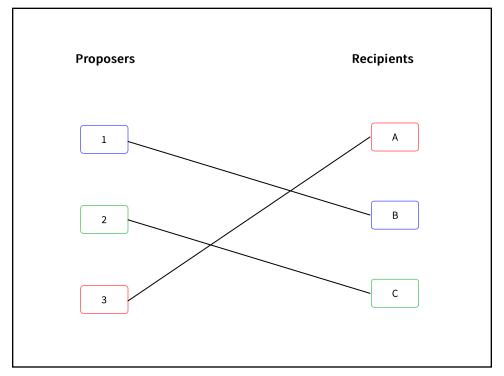
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Is there always a perfect matching that's stable?

- Yes, Gale-Shapley algorithm (1962)
- Deferred acceptance algorithm



Demo https://gale-shapley.com				
Proposers	Recipients			
A B C	3 2 1 A			
A C B 2	B 2 1 3			
A B C 3	1 3 2 c			



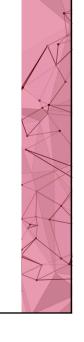
Gale-Shapley algorithm: Men-proposing version

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assign each person to be free; while some man m is free do begin w:= first woman on m's list to whom m has not yet proposed; if w is free then assign m and w to be engaged \{to each other\}else if w prefers m to her fiancé m' then assign m and w to be engaged and m' to be free else w rejects m \{and m remains free\}end; output the stable matching consisting of the n engaged pairs
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Gale-Shapley algorithm

- Thm 1.2.1. The algorithm terminates with a stable matching.
- Thm 1.2.2. Men-proposing version is men-optimal [ordering of men doesn't matter]
- Thm 1.2.3. Men proposing version is the worst for women [each woman gets the worst man <u>subject to the matching being stable</u>]



Propos	sers	Recipients
a b 1		a 1
b a 2	Best for proposers	b 2
	Worst for recipients	
F	Proposers R	ecipients
	1	а
	2	b

Proposers		Recipients
2 1 a		a b
1 2 b		b a
	Best for proposers Worst for recipients	
Propo	sers Reci	pients
а		2
Ь		1



Applications

beyond kidney exchange

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Residency matching



Hospitals interview candidates and rank them



Candidates rank hospitals that interviewed them





NYC high school matching

- Around 80K 8-th graders are matched to around 500 high schools
- Each student ranks at most 12 schools
- Schools rank applicants
 - 'But schools continue to tell parents and students "with a wink" that
 they may be penalized if they don't list their school first.'
 (https://www.dnainfo.com/new-york/20161115/kensington/nyc-high-school-admissions-ranking)
- Match by DOE



Content delivery networks (CDN)

Algorithmic Nuggets in Content Delivery

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ABSTRACT

This paper "peeks under the covers" at the subsystems that provide the basic functionality of a leading content delivery network. Based on our experiences in building one of the largest distributed systems in the world, we illustrate how sophisticated algorithmic research has been adapted to balance the load between and within server clusters, manage the caches on servers, select paths through an overlay routing network, and elect leaders in various contexts. In each instance, we first explain the theory underlying the algorithms, then introduce practical considerations not captured by the theoretical models, and finally describe what is implemented in practice. Through these examples, we highlight the role of algorithmic research in the design of complex networked systems. The paper also illustrates the close synergy that exists between research and industry where research ideas cross over into products and product requirements drive future research.

1. INTRODUCTION

The top-three objectives for the designers and operators of a content delivery network (CDN) are high reliability, fast and consistent performance, and low operating cost.



Figure 1: A CDN serves content in response to a client's request.

CDN's authoritative name server. The authoritative name